Parameter Extraction for Symmetric Coupled-Resonator Filters

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***Abstract* — A new parameter extraction procedure for symmetric coupled-resonator filters is presented. Closed form recursive formulas are derived for the synthesis of all the filter parameters from known zeros and poles of the even- and odd-mode input impedance functions. The simple and straightforward procedure can eliminate complicated optimization routines and have extensive applications in design and tuning of filters.**

I. INTRODUCTION

With the rapid growth of communication industry, the need for high-performance microwave filters which possess optimum responses consistent with minimum weight and compact size is apparent. Furthermore, the optimum filters must have the maximum possible number of finite transmission zeros for selectivity or phase linearity considerations. Among all the possible filter configurations, coupled resonator filter in a symmetric folded configuration (the canonical form) is one of the preferable candidates [1].

The knowledge of the parameters of such high performance filters is very important since highly accurate couplings and resonant frequencies are required to ensure the desired responses. Parameter extraction methods have been extensively studied for the design and tuning of such filters. Thal [2] developed a method that incorporated equivalent-circuit analysis programs with element optimization routines. Accatino [3] utilized phase measurement of the input admittance of a short-circuited filter in conjunction with LCX synthesis and minimum pattern search optimization techniques to extract the filter parameters. Various optimization routines have played important roles for traditional approaches in parameter extractions. The optimization variables are either network element values (for model-based approaches) [2,4] or physical dimensions of the filters [6]. Sensitivity analysis of the optimization variables on the final responses has been performed and much effort has been devoted to improve the convergence of the optimization routines in all cases [4-7].

While using optimization may be an attractive approach, it is usually inefficient and quite complicated.

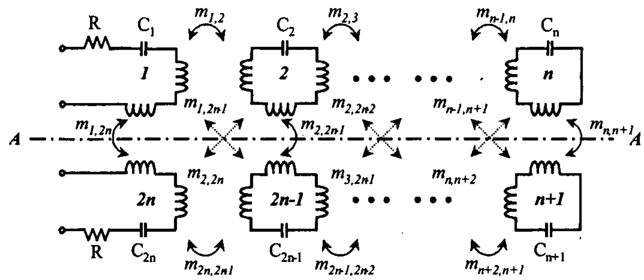

Fig. 1 Equivalent circuit representation for the symmetric coupled-resonator filters.

Also, it depends strongly on the initial guess of the optimization variables and often results in non-optimum (local minimum) solutions. In this paper, a simple and straightforward parameter extraction procedure for symmetric coupled-resonator filters is presented. Closed form recursive formulas are derived which synthesize all the filter parameters from the zeros and poles of the even- and odd-mode input impedance functions of the network. Equivalent circuit representation and circuit analysis of the symmetric coupled-resonator filters are included in Section II. Section III deals with the detailed derivation of the extraction procedure. The design example presented in Section IV shows the powerfulness of the proposed procedure.

II. CIRCUIT MODEL AND ANALYSIS

Fig. 1 shows the equivalent circuit representation of the filters realized in canonical form. Each resonator is composed of a series capacitance C_i ($i=1,...2n$) together with total loop inductance L_i. Couplings between resonators are represented by the frequency independent reactance $m_{i,j}$ and R is the equivalent input/output coupling resistance, respectively. For canonical filters, in general, cascade (or series) couplings of the same sign are provided between consecutively numbered cavities and shunt (or cross) couplings of arbitrary signs are provided between cavities *1* and *2n*, *2* and *2n-1*, ..., etc. [1]. Couplings between cavities *1* and *2n-1*, *2* and *2n*,..., etc. may also exist (as unwanted but unavoidable) due to the very compact structure in this configuration. The following relations hold when $\overline{AA'}$ is the plane of symmetry (Fig. 1) :

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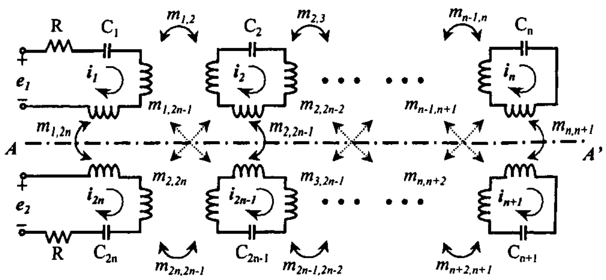

Even Excitation : $e_2=e_1$, $i_1=i_{2n}$; $i_2=i_{2n-1}$; ... $i_n=i_{n+1}$, PMC at AA'
Odd Excitation : $e_2=-e_1$, $i_1=-i_{2n}$; $i_2=-i_{2n-1}$; ... $i_n=-i_{n+1}$, PEC at AA'

Fig. 2 Even and odd excitations of the original network. Each excitation will result in a singly-terminated bisected network.

$$m_{i,i+1} = m_{(2n-i),(2n+1-i)} \ , \ i = 1,2,\dots n-1 \tag{1a}$$

$$m_{i,(2n-i)} = m_{(i+1),(2n+2-i)} \ , \ i = 1,2,\dots n-1 \tag{1b}$$

The loop equations of the structure in matrix form can be written as [5]:

$$\bar{e} = j(\lambda[I] - [M])\bar{J} \tag{2}$$

In the above expression, $[M]$ is the coupling matrix with the diagonal elements denoting the offset in resonant frequencies of each individual resonator and the normalized frequency λ is defined as:

$$\lambda(f) = \frac{f_o}{BW}\left(\frac{f}{f_o} - \frac{f_o}{f}\right) \tag{3}$$

where f_o and BW are the center frequency and bandwidth of the filter, respectively.

Analysis of the structure is most easily accomplished using bisection to obtain the singly terminated even- and odd-mode networks as shown in Fig.2. The input impedance at loop i of the even- and odd-mode singly terminated networks can readily be derived as [8]:

$$Z^{(i)}_{in<e,m>}(\lambda) = jA\frac{P_{i<e,m>}(\lambda)}{Q_{i<e,m>}(\lambda)}, \ i=1,2,\dots,n \tag{4}$$

where A is the normalization constant and the monic polynomials P and Q can be expressed as:

$$P_{i<e,m>}(\lambda) = \sum_{t=0}^{n-i+1} c^{(i)}_{<e,m>t}\lambda^{t} = \prod_{t=1}^{n-i+1}(\lambda - \lambda^{(i)}_{z<e,m>t}), i=1,\dots,n \tag{5}$$

$$Q_{i<e,m>}(\lambda) = \sum_{q=0}^{n-i} d^{(i)}_{<e,m>q}\lambda^{q} = \prod_{q=1}^{n-i}(\lambda - \lambda^{(i)}_{p<e,m>q}), i=1,\dots,n \tag{6}$$

where $\lambda^{(i)}_{z<e,m>t}$ and $\lambda^{(i)}_{p<e,m>q}$ are the normalized zero and pole frequencies of $P_{i<e,m>}(\lambda)$ and $Q_{i<e,m>}(\lambda)$, corresponding to the normalized zero and pole frequencies of the input impedances of the bisected even- or odd-mode network at loop i, respectively. The input reflection coefficient for the bisected networks can be expressed as:

$$S_{11<e,m>} = \frac{Z^{(1)}_{in<e,m>} - R}{Z^{(1)}_{in<e,m>} + R} \tag{7}$$

Finally, the two-port scattering parameters for the symmetric network are then :

$$S_{11} = S_{22} = 0.5(S_{11m} + S_{11e}) \tag{8a}$$

$$S_{21} = 0.5(S_{11m} - S_{11e}) \tag{8b}$$

III. Parameter Extraction Procedure

The parameter extraction procedure can be summarized as follows:

(1) Computation of the input reflection coefficients S_{11e} and S_{11m}. This can be done either by direct computation through EM simulation with proper boundary conditions imposed at plane of symmetry (PEC for S_{11e} and PMC for S_{11m}) or by deriving from the total (simulated or measured) network response as:

$$S_{11m} = (S_{11} + S_{21}) \tag{9a}$$

$$S_{11e} = (S_{11} - S_{21}) \tag{9b}$$

(2) Determine the zeros and poles of the bisected networks from S_{11e} and S_{11m}. In general, the frequencies corresponding to $\pm 180^\circ$ and 0° phases of the input reflection coefficient are the corresponding zeros and poles of the networks. Accurate determination of zeros and poles may require possible reference plane adjustment. For a filter with $2n$ resonators, there are n zeros and $(n-1)$ poles for each bisected network.

(3) Once the zeros and poles are known, the center frequency f_o of the filter can be obtained by numerically solving the following equation:

$$\frac{\left[\prod_{t=1}^{n-i+1}\left(f^{m}_{zt} - f_o\right)\right]\left[\prod_{q=1}^{n-i}\left(f^{e}_{pq} - f_o\right)\right]}{\left[\prod_{t=1}^{n-i+1}\left(f^{e}_{zt} - f_o\right)\right]\left[\prod_{q=1}^{n-i}\left(f^{m}_{pq} - f_o\right)\right]} = \frac{\tan\left(\frac{\theta_m(f_o)}{2}\right)}{\tan\left(\frac{\theta_e(f_o)}{2}\right)} \tag{10}$$

where $\theta_m(f)$ and $\theta_e(f)$ are the phase responses of S_{11m} and S_{11e}; $f^{m,e}_{zt}$ and $f^{m,e}_{pq}$ are the corresponding zeros and poles, respectively.

(4) The offset in resonant frequencies of each resonator and the coupling coefficients can be synthesized from the following closed-form recursive relations:

$$m_{ii} = -\frac{1}{2}\left[c^{(i)}_{m(n-i)} - d^{(i)}_{m(n-i-1)} + c^{(i)}_{e(n-i)} - d^{(i)}_{e(n-i-1)}\right] \tag{11}$$

$$m_{i,(2n+1-i)} = -\frac{1}{2}\left[c^{(i)}_{m(n-i)} - d^{(i)}_{m(n-i-1)} - c^{(i)}_{e(n-i)} + d^{(i)}_{e(n-i-1)}\right] \tag{12}$$

Define:

$$A^{(k)^2}_{m} \equiv \left(c^{(k)}_{m(n-k)} - d^{(k)}_{m(n-k-1)}\right)d^{(k)}_{m(n-k-1)} - c^{(k)}_{m(n-k-1)} + d^{(k)}_{m(n-k-2)} \tag{13a}$$

$$A^{(k)^2}_{e} \equiv \left(c^{(k)}_{e(n-k)} - d^{(k)}_{e(n-k-1)}\right)d^{(k)}_{e(n-k-1)} - c^{(k)}_{e(n-k-1)} + d^{(k)}_{e(n-k-2)} \tag{13b}$$

$$m_{k,k+1} = \frac{1}{2}(A^{(k)}_{m} + A^{(k)}_{e}) \tag{14}$$

$$m_{k,(2n-k)} = \frac{1}{2}(A^{(k)}_{m} - A^{(k)}_{e}) \tag{15}$$

$$P_{k+1<e,m>}(\lambda) = Q_{k<e,m>}(\lambda) \tag{16}$$

$$P_{k<e,m>}(\lambda) = P_{k+1<e,m>}(\lambda)[\lambda + m_{k,k} \pm m_{k,2n+1-k}] - Q_{k+1<e,m>}(\lambda)[m_{k,k+1} \pm m_{k,2n-k}]^2 \quad (17)$$

with $i=1,2...,n$ and $k=1,2,...,n-1$; c's and d's are the coefficients as specified in Eqn.(5)&(6).

(5) Finally, the input/output equivalent coupling resistance can be calculated as

$$R = -\frac{\prod_{t=1}^{n}(\lambda_{90} - \lambda_{zmt}^{(1)})}{\prod_{q=1}^{n-1}(\lambda_{90} - \lambda_{pmq}^{(1)})} \quad (18)$$

where λ_{90} is the normalized frequency corresponding to $\pm 90^{\circ}$ phase of the bisected even-mode network.

A computer program has been developed to perform the parameter extraction based on the above procedure. It is clear that this procedure provides a simple and straightforward way to synthesize all the filter parameters from the known zeros and poles of the bisected networks. The explicit recursive relations presented here yield accurate and unique solutions which eliminate the complexity and inefficiency of optimization routines.

IV. Design Example

The proposed procedure has been extensively tested through many examples and has been proven to be accurate and powerful. To demonstrate its feasibility, the procedure is applied in designing a 4-pole elliptic function filter with 30 MHz bandwidth centered at 3 GHz. The filter is intended to be realized in microstrip structure for high-temperature-superconductivity (HTS) applications. The synthesized prototype filter has the following input/output equivalent coupling resistance and coupling matrix:

$R_{in} = R_{out} = 1.2535$

$$M = \begin{bmatrix} 0.0000 & 0.9799 & 0.0000 & -0.1095 \\ 0.9799 & 0.0000 & 0.7875 & 0.0000 \\ 0.0000 & 0.7875 & 0.0000 & 0.9799 \\ -0.1095 & 0.0000 & 0.9799 & 0.0000 \end{bmatrix}$$

Fig. 3 shows the ideal circuit response of the prototype filter. An unloaded Q of 50,000 is used in calculation.

In order to minimize the overall filter size, the resonators are realized in capacitively-loaded hairpin-comb structure [9]. Electric (negative) coupling is achieved through the open ends of the resonators and tap-in configuration is adapted for input/output coupling.

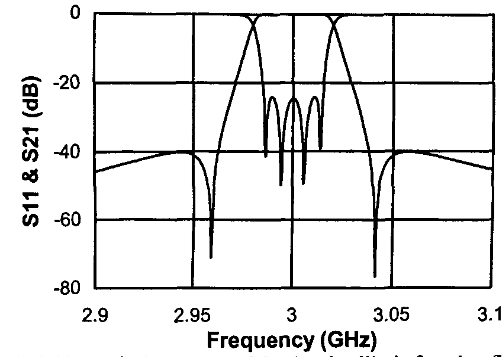

Fig.3 Theoretical response of the 4-pole elliptic function filter.

Fig. 4 shows the layout of the filter on a 20 mil thick MgO substrate with ε_r=9.8and tanδ=5×10^{-6}. The corresponding spacings between the resonators are determined through the characterization of the couplings as described in [10].

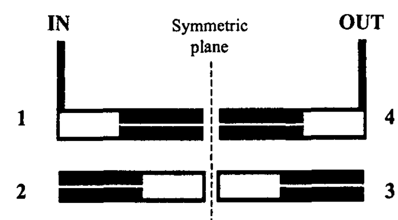

Fig. 4 Layout of the 4-pole filter on MgO substrate

The whole filter structure is simulated using commercial EM simulation software package Zeland™ [11]. To save the computation time, half of the structure is simulated twice by putting PEC and PMC at the plane of symmetry. Fig. 5 shows the simulation result. The phase responses of S_{11e} and S_{11m}, after proper reference plane adjustment, are plotted in Fig. 6 showing the corresponding zero and pole frequencies. The following parameters are then extracted:

$R_{in} = R_{out} = 1.3320 \quad f_o = 3.0005$ GHz

$$M_{extract} = \begin{bmatrix} -0.1055 & 1.1545 & -0.076 & -0.1054 \\ 1.1545 & -0.0400 & 0.7688 & -0.0760 \\ -0.0760 & 0.7688 & -0.0400 & 1.1545 \\ -0.1054 & -0.076 & 1.1545 & -0.1055 \end{bmatrix}$$

The calculated responses using the above extracted parameters are included in Fig. 5. Good agreement is observed. The asymmetric levels of the flyouts are believed to be caused by the spurious cross coupling m_{13} (m_{24}) due to the tightly-spaced resonators. The value of m_{13} can be accurately predicted by this procedure. The existence and sign of m_{13} can be further confirmed through the additional simulation of a 2-port subset composed of resonators 1,2,3 (basically Chebyshev type-

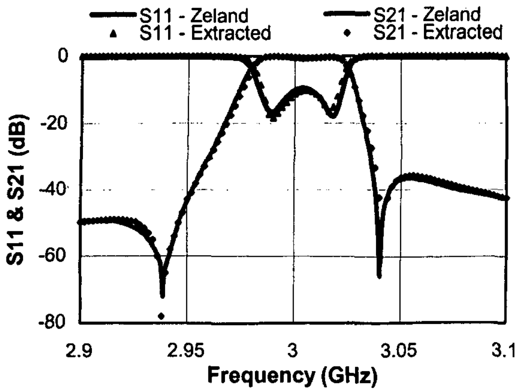

Fig. 5 EM simulated responses and responses using extracted parameters.

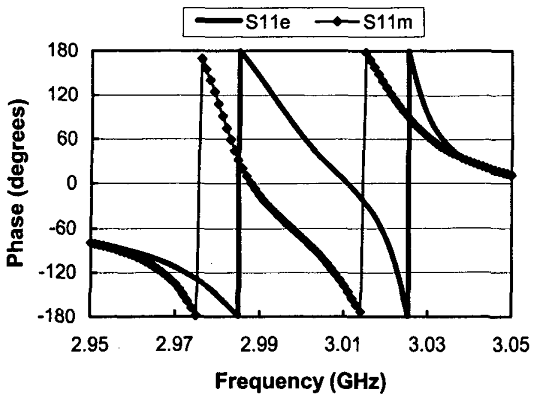

Fig. 6 Phase responses of S_{11e} and S_{11m}

coupled resonators) from the original structure. The transmission zero shown in the result (Fig.7) proves the existence of negative m_{13}. Fig. 8 shows the final simulated response after adjustment with final extracted parameters listed as:

$$R_{in} = R_{out} = 1.2835 \qquad f_o = 2.9999 \text{ GHz}$$

$$M_{final} = \begin{bmatrix} -0.060 & 0.9899 & -0.075 & -0.1085 \\ 0.9899 & 0.01 & 0.7872 & -0.075 \\ -0.075 & 0.7872 & 0.01 & 0.9899 \\ -0.1085 & -0.075 & 0.9899 & -0.060 \end{bmatrix}$$

V. CONCLUSION

A new and powerful parameter extraction procedure has been presented. Closed form recursive formulas are given for the synthesis of all filter parameters trough the knowledge of the zeros and poles of corresponding bisected networks. This procedure has been proven feasible through design examples and can also be applied in filter measurements, diagnosis and tuning.

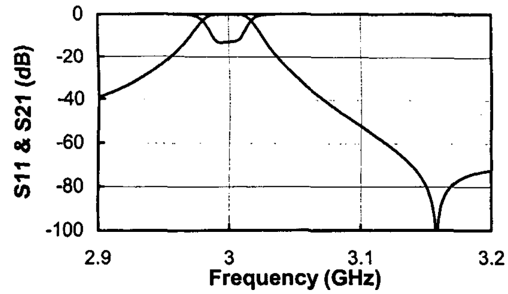

Fig. 7 EM simulated result of the 3-pole subset showing the existence of m_{13}.

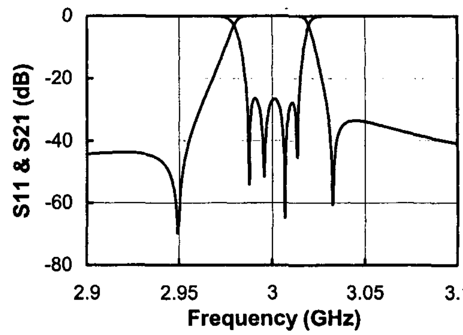

Fig. 8 EM simulated final response after adjustment.

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